Solvency Tests for Groups

FINMA, Quantitatives Risikomanagement
Michael Schmutz
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Introduction: Consolidated, Group Structure Model, Granular

Set-valued risk measures ⇒ a set valued point of view and explanations
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Set-valued risk measures \(\Rightarrow\) a set valued point of view and explanations

FOPI (several presentations) for further discussions and developments.

Filipović & Kupper (2008) consider optimal capital and risk transfers for group diversification.

Keller (2014) puts group solvency tests in a broader setting.

... 

A link of some aspects of the above references to an active area of research (set-valued risk measures) will be sketched today. Before some of the most important statements of the above references will be recalled.

**Important Remark:** FINMA does not use set-valued risk measures.
Approaches to Insurance Groups: Example of a Simple Setting

Parent Company Pa

Subsidiary S1

Subsidiary S2

Subsidiary S3

Policy holders
Is being part of a group favourable or unfavourable?

- For a given legal entity (e.g. S1)?
- For stake holders of the company?
- In particular for the policyholders of that legal entity?

⇒ How shall group effects be treated?
Different and sometimes contradictory answers exist:

1. “A capital requirement for the individual insurance company can be abolished. It is replaced by a single capital requirement for the group.” ⇔ Consolidated approach.

2. “The capital requirement for the insurance company is determined as if the group did not exist. This means for example that internal reinsurance cover cannot be taken into account”.

3. “Being part of a group is dangerous. The capital requirement for the insurance company from (2) has to be increased by 30%.”

4. “Being part of a group has many effects on the insurance company. Some of these effects increase the risk, some decrease it. All effects have to be taken into account in the calculation of the capital requirements of the group members. A group wide capital requirement is not necessary.” ⇔ Legal entity approach.
Problematic point of view: The consolidated approach

- Considers the group as one single economic entity.
- Is based on a (consolidated) balance sheet of all assets and liabilities of all legal entities of the group (ignoring in which entity the positions are held).
- Intragroup transactions cancel out.
Hence, if a group consists of \( n \) legal entities with \( C_i \) being the available capital\(^1\) of legal entity \( i \in \{1, \ldots, n\} \) then

\[
C = \sum_{i=1}^{n} C_i
\]

has to be an acceptable random variable with respect to the relevant risk measure \( \rho \), i.e.

\[
\rho \left( \sum_{i=1}^{n} C_i \right) \leq 0.
\]  \hspace{1cm} (1)

Later we will clearly come back to some of the well-known problems related to this approach.

\(^1\) in particular without shareholdings of other legal entities of the same group.
A legal entity approach: call it “granular”

Idea: It is *not* in the responsibility of an insurance group but of individual legal entities to pay for claims of policy holders. This is particularly relevant under *stress*.

In view of that:

- **Focus** is on the **legal entities**.
- The existence of the **group** has an **impact** on individual legal entities.
  - Therefore they have to be considered as members of the group instead of as standalone entities.
  - Effects of the group on individual entities are part of the model (taken into account in the form of capital and risk transfer instruments (CRTI) and of the ownership relations within the group).

⇒ This is why such a legal entity approach is a **group model** (joint modelling of the group’s legal entities is required).

⇒ It is fundamentally more than a collection of traditional solo supervisions.
If $C_i$ now stands for the available capital after the relevant time period and after (valuation of) CRTI (what is often very complicated to derive due to complicated interdependencies):

$$\rho(C_i) \leq 0$$

for all $i \in \{1, \ldots, n\}$ is required.

In the SST CRTI are written, legally binding and enforceable contracts that define

- in which situation
- how much capital and/or risk
- flow between which legal entities

Examples of CRTI:

- Reinsurance agreements
- Financial guarantees
- Hybrid instruments
- Intragroup loans
“Intended transactions”

Possible, intended, transactions or the oral promise from a group CEO to support subsidiaries in distressed situations is not a CRTI.

The SST requires a granular approach. In addition a consolidated view might be permitted by FINMA either upon application by the group or requested by FINMA (Appendix 2, FINMA circular 08/44).
Some groups consist of a large number of legal entities. In such cases, modelling them on an individual basis is considered too complicated. In view of that under some conditions FINMA may grant upon application that some of them can be merged into clusters as a simplification.
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Set-valued risk measures ⇒ a set valued point of view and explanations
Results for set-valued risks

- Ben Tahar & Lepinette (2014) and Feinstein & Rudloff (2012, 2013) worked out the dynamical setting.
- Farkas, Koch-Medina, Munari (2014) investigate the link between multi-asset risk measures and set valued risk measures as defined by Hamel, Heyde & Rudloff (2011). Apply multi-asset risk measures to optimal risk sharing amongst several lines of business.
- Exact calculation of risks is a rather complicated numerical procedure. New algorithms are being developed (Löhne et al.).
- Haier, Molchanov, MS (University of Berne, work in progress) adaption and application of Cascos & Molchanov (2014) to measuring group risks (*rough idea of the concept will be presented on the following slides*).
Kabanov’s exchange cone model

- $X$ is a $L^p$-integrable random vector in $\mathbb{R}^d$ representing a financial outcome.
- $K$ is the random exchange cone.
- $K$ describes transaction rules at the time when $X$ is assessed ($K$ is the cone of portfolios available at price zero).

Aim: measure the risk of $X$ taking $K$ into account.
Set-valued risk measures have been introduced with the aim to handle exchanges with transaction costs and random exchange rates. We first describe this setting before sketching their use in relation to solvency tests.

- **Idea:** consider the risk of $X = X + K$
- The portfolio $X$ is a convex closed set such that $X = X + \mathbb{R}^d$ (lower set).
- Risk $\rho(X)$ is an upper set.

- **Key idea:** A portfolio $X$ is acceptable if there exists a random vector $\xi \in X$ a.s. so that all components of $\xi$ are individually acceptable.
If $K$ represents some transaction costs:

Note: Although components of $\xi$ are considered separately, $\xi$ itself appears as a linear combination of components of $X$ and $K$, so that the results are not marginalised.
• A random vector $\xi \in L^p(\mathbb{R}^d)$ is called a selection of $X$ if $\xi \in X$ a.s. (assume that $X$ has at least one $p$-integrable selection, that is $L^p(X)$ is not empty).

• Consider $d$-tuple $r = (r_1, \ldots, r_d)$ of univariate coherent law invariant $L^p$-risk measures.

• The random set $X$ is acceptable if it possesses at least one acceptable selection $\xi$, meaning that

$$r(\xi) = (r_1(\xi_1), \ldots, r_d(\xi_d)) \leq 0,$$

i.e. all individual coordinates of $\xi$ are acceptable.
- The *selection risk measure* $\rho_s(X)$ is the closure of the set $\rho_{s,0}(X) = \{ a \in \mathbb{R}^d : X + a \text{ is acceptable} \}$.

- Equivalently
  \[ \rho_s(X) = \text{cl} \bigcup_{\xi \in L^p(X)} [r(\xi), \infty) \].

A portfolio is **acceptable** means that the risk measure **contains the origin** (lower risks are considered).
Selection risk measures have good properties: Cascos & Molchanov

Following Cascos & Molchanov (2014) selection risk measures have good properties, e.g.

- law invariant
- \( \rho_s(X + a) = \rho(X) - a \) for all \( a \in \mathbb{R}^d \) (cash invariance).
- If \( X \subset Y \) a.s., then \( \rho_s(X) \subset \rho_s(Y) \) (monotonicity, which is very important in the context of solvency tests for groups)
- \( \rho_s(cX) = c\rho_s(X) \) for all \( c > 0 \) (positive homogeneity)
- \( \rho_s(X + Y) \supset \rho_s(X) + \rho_s(Y) \) (superadditivity for inclusion)
- \( \rho_s \) admits a dual representation.

Superadditivity for inclusion corresponds to the subadditivity of risks — the “crucial” property of coherent risk measures.
For simplicity we assume that there are only two legal entities in the group. The available capital amounts are given by $C = (C_1, C_2)$, where $C_i \in L^p$, $i \in \{1, 2\}$ and $r_1 = r_2$. Assume first that there are no transaction costs (for moving capital). Consider the subspace $U$ and the affine subspace $H$ given by

$$U = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}, \quad H = C + U$$

and

$$X = H + \mathbb{R}^2_-.\]
Then we have that $C_1 + C_2$ is acceptable if and only if $X$ is acceptable with respect to $\rho_s$.

This formalises and/or reformulates interpretations which were stated more heuristically several times in the past (in particular on slides from the FOPI), e.g.

- Assumes that any asset can legally be used to cover any loss.
- Assumes that the top management is willing to use / spend any asset to cover any loss.
- Assumes that group acts as one single economic entity even in times of distress.
- Etc.

⇒ The consolidated approach is not reasonable (even as a supplement).
⇒ Even for supplementary purposes the exchange cone has to be adapted.
The granular approach technically translates to a setting with infinite transaction costs, namely

\[ \mathbf{X} = \mathbf{C} + \mathbb{R}_d \]

needs to be acceptable. If \( \mathbf{C} \) denotes capital amounts before any intragroup transactions we first have to modify it to a vector \( \tilde{\mathbf{C}} \) including intragroup transactions.

In this case

\[ \tilde{\mathbf{X}} = \tilde{\mathbf{C}} + \mathbb{R}_d \]

has to be acceptable.
Intermediate cases

The granular approach with clusters can be obtained by suitably combining the two approaches.

The granular solvency test (set valued version) or the granular solvency test with clusters (set valued versions) can be obtained from the consolidated approach by restricting the exchange cone.

Probably the main advantage of a set valued approach is the possibility to combine a financial outcome $C$ with suitable transfers (represented by a set).

Of course, “suitable” does not need to be the same for everyone (policyholder, regulator vs. shareholder).
Since the consolidated approach includes transfers which are legally not attainable and also transfers which are, from an economical point of view, not reasonable, we obtain that the exchange cone has to be restricted even for a reasonable supplement.

Possible (but perhaps not reasonable):

- Exclude all illegal transfers
- Exclude transfers which ignore the LLPO
- ...

In many cases it is not possible anymore to work with a fixed exchange cone $K$ (exchange sets with shape depending on the financial outcome need to be applied / set of possible transactions becomes smaller under stress), which has many interesting consequences.
In a set-valued approach many important aspects can (partially) be naturally taken into account, e.g.

- Some foreign currencies aspects via “random slopes”.
- Fungibility costs and restrictions (e.g. related to Lloyd’s business) via change of the “upper bound”.
- ...
Selected References