Risk Measures when Changing the Calculation Currency

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I heard we don't pass the solvency test...

Now we do: I changed the calculation currency!
Internal Models in a Multi Currency Setup

Relevant currencies in an Internal Model:

• underlying currency:
  – currency in which a balance sheet position is denominated

• calculation currency:
  – for aggregation purposes, the exposures denominated in the various currencies need to be converted into a common currency

• reporting currency:
  – currency chosen to report risk figures
Converting to the Calculation Currency

For simplicity we consider a balance sheet with only two currencies:

*domestic currency* : $d$

*foreign currency* : $f$

which are converted into an aggregation currency $a$.

To calculate the capital position at time $T$, we need the corresponding (random) exchange rates at time $T$: $e_{d,a}$, $e_{f,a}$

The capital position at time $T$, expressed in currency $a$, is given by

$$\hat{X}_a := [A_d - L_d] \cdot e_{d,a} + [A_f - L_f] \cdot e_{f,a}$$

where $A$ and $L$ stand for the Assets and Liabilities in the respective currency.
Observations

- Required Solvency Capital - calculated as TailVar or Var - depends on the calculation currency

- As a consequence, the Solvency Ratio (required capital / available capital) also depends on the calculation currency

What about Capital Adequacy Tests?
Capital Adequacy Tests

VaR based:

\[ P(X < 0) \leq \alpha \]

Corresponds to testing whether the probability that the insurer will not be able to meet its obligations is below \( \alpha \).
Example: Solvency II (\( \alpha = 1/200 \))

Expected Shortfall based:

\[ ES_{\alpha}(X) \leq 0 \]

Roughly speaking, it corresponds to testing whether in the worst \( \alpha \times 100 \) percent states, the company will be able to meet its obligations on average.
Example: SST (\( \alpha = 1/100 \))
Currency Invariance of Capital Adequacy Tests

VaR based capital adequacy tests are currency invariant, i.e.

\[ P(X_d < 0) \leq \alpha \iff P(e_{d,f} X_d < 0) \leq \alpha \]

But the analogous result does not hold for Expected Shortfall based capital adequacy test!

This was first observed by Artzner, Delbaen & Koch-Medina in 2009 (ASTIN Bulletin 39), but the result got largely unnoticed in the risk management community.
Example

- Discrete Setup (e.g. "simulation approach"): 1000 possible states of the world, each with the same probability
- The worst 1% states (in term of economic value), expressed in the domestic currency are given by

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<thead>
<tr>
<th>State</th>
<th>( \omega_{10} )</th>
<th>( \omega_9 )</th>
<th>( \omega_8 )</th>
<th>( \omega_7 )</th>
<th>( \omega_6 )</th>
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<tr>
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\[ ES_{0.01}(X_d) = -5 < 0 \rightarrow \text{Company is solvent} \]
Example (cont)

Exchange rate model: \( e_{d,f}(\omega_1) = e_{d,f}(\omega_2) = 2, \quad e_{d,f}(\omega_i) = 1, \forall i > 2 \)

i.e. the exchange rate is 1, except in the two worst outcomes where we observe a strong devaluation of the foreign currency vs. the domestic currency.

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<tr>
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<td>-20</td>
<td>-60</td>
<td>-80</td>
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\[ ES_{0.01}(X_f) = 2 > 0 \rightarrow \text{Company is insolvent} \]
Is this relevant?

• For our example we needed a model where a currency depreciates at the time where the worst economic losses happen.

• We have evidence from history that such events are not implausible:
  – in the 2008 financial crisis, the US Dollar appreciated against almost all currencies
  – after the Tohuku Earthquake the Japanese Yen depreciated for a short period and started appreciating a month later at a very rapid rate
References


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